The impact of group interactions on opinion formation

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XLIV Dynamics Days Europe 2024 – Minisymposium: Adaptive dynamical networks

Slides

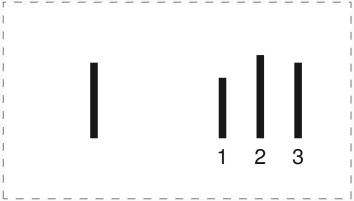
GROUP DYNAMICS IN OPINION FORMATION

Opinions are seldomly formed in isolation, but rather through **interactions** with others in society.

Besides pairwise interactions between individuals, **group dynamics** may play an integral part in opinion formation due to each individual's striving for conformity (bandwagon effect, peer-pressure ...).

Source: cottonbro studio

GROUP DYNAMICS IN OPINION FORMATION



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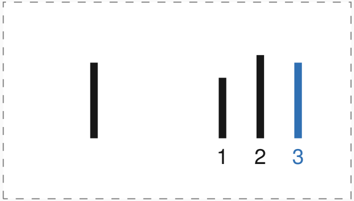
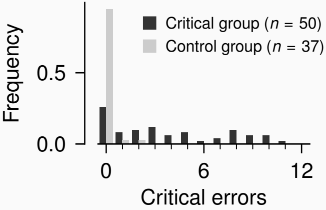
GROUP DYNAMICS IN OPINION FORMATION



Source: cottonbro studio

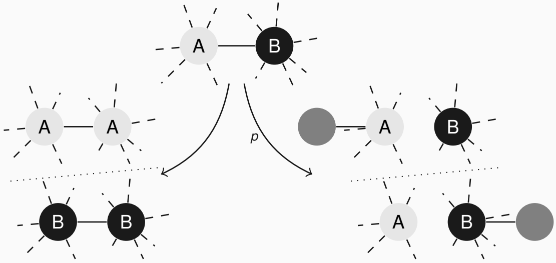
GROUP DYNAMICS IN OPINION FORMATION



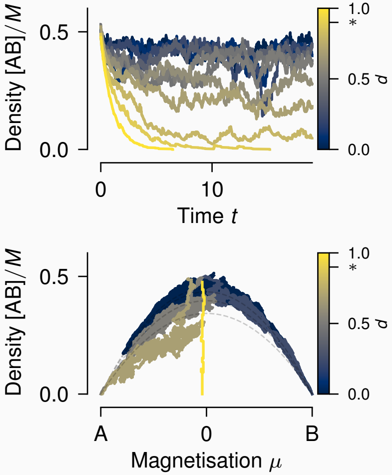
"The tendency to conformity in our society is so strong that reasonably intelligent and well-meaning young people are willing to call white black." *(Solomon Asch, 1955)*

Source: cottonbro studio

CLASSICAL ADAPTIVE VOTER MODEL



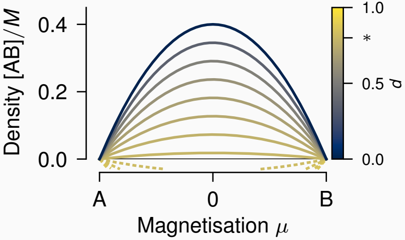
Adaptivity: rewire-to-same or rewire-to-random.

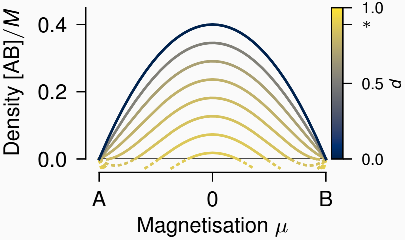


CLASSICAL ADAPTIVE VOTER MODEL: MEAN-FIELD DESCRIPTION

$$
\begin{cases}
[\dot{\mathrm{A}}] = 0 \\
[\dot{\mathrm{B}}] = 0 \\
[\mathrm{A}\dot{\mathrm{A}}] = \frac{1-\pi_{\mathrm{A}\to\mathrm{B}}\,p}{2}[\mathrm{AB}] + \frac{1-p}{2}\left(2\,[\mathrm{A(B)A}] - [\mathrm{A(A)B}]\right) \\
[\mathrm{A}\dot{\mathrm{B}}] = -\frac{2-(\pi_{\mathrm{A}\to\mathrm{B}}+\pi_{\mathrm{B}\to\mathrm{A}})\,p}{2}[\mathrm{AB}] - \frac{1-p}{2}\left(2\,[\mathrm{A(B)A}] - [\mathrm{A(A)B}] - [\mathrm{A(B)B}] + 2\,[\mathrm{B(A)B}]\right) \\
[\mathrm{B}\dot{\mathrm{B}}] = \frac{1-\pi_{\mathrm{B}\to\mathrm{A}}\,p}{2}[\mathrm{AB}] + \frac{1-p}{2}\left(2\,[\mathrm{B(A)B}] - [\mathrm{A(B)B}]\right)
\end{cases}
$$

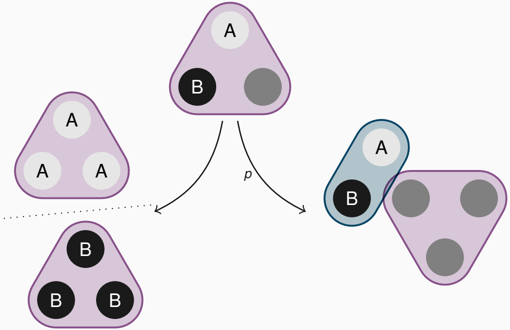
with $\pi_{\mathrm{X}\to\mathrm{X}'} \propto \delta_{\mathrm{X},\mathrm{X}'}$ for rewire-to-same and $\pi_{\mathrm{X}\to\mathrm{X}'} \propto [\mathrm{X}']$ for rewire-to-random adaptivity.





HYPERGRAPH ADAPTIVE VOTER MODEL

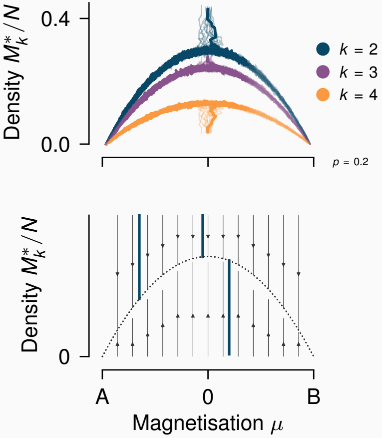
We use a **hypergraph** to encode polyadic interactions.



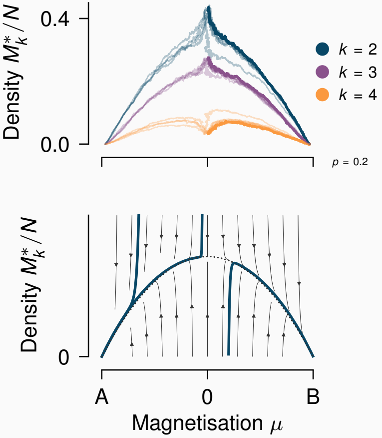
Adaptivity: rewire-to-same or rewire-to-random.
Propagation: proportional voting or majority voting.

HYPERGRAPH ADAPTIVE VOTER MODEL: DYNAMICS

Proportional voting



Majority voting



HYPERGRAPH ADAPTIVE VOTER MODEL: MEAN-FIELD DESCRIPTION

$$
\left\{
\begin{aligned}
[\dot{\mathrm{A}}] &= (1-p)\Bigg[\sum_{\substack{(\hat a,\hat b)\in\mathbb{N}^2\\ \hat a+\hat b\le K}} [\mathrm{A}^{\hat a}\mathrm{B}^{\hat b}](\hat b\,\eta_{\mathrm{A}}(\hat a,\hat b) - \hat a\,\eta_{\mathrm{B}}(\hat a,\hat b))\Bigg]\\
[\dot{\mathrm{B}}] &= (1-p)\Bigg[\sum_{\substack{(\hat a,\hat b)\in\mathbb{N}^2\\ \hat a+\hat b\le K}} [\mathrm{A}^{\hat a}\mathrm{B}^{\hat b}](\hat a\,\eta_{\mathrm{B}}(\hat a,\hat b) - \hat b\,\eta_{\mathrm{A}}(\hat a,\hat b))\Bigg]\\
[\dot{\overline{\mathrm{A}^{a}\mathrm{B}^{b}}}] &= p\Bigg[\frac{a+1}{a+b+1}[\mathrm{A}^{a+1}\mathrm{B}^{b}]\,\mathbb{1}_{\mathbb{N}^2\cap\bar B^1_K(0)}(a+1,b) + \frac{b+1}{a+b+1}[\mathrm{A}^{a}\mathrm{B}^{b+1}]\,\mathbb{1}_{\mathbb{N}^2\cap\bar B^1_K(0)}(a,b+1) - [\mathrm{A}^{a}\mathrm{B}^{b}]\,\mathbb{1}_{\mathbb{N}^2\cap\bar B^1_K(0)}(a,b)\\
&\quad + \sum_{\substack{(\hat a,\hat b)\in\mathbb{N}^2\\ \hat a+\hat b\le K}} [\mathrm{A}^{\hat a}\mathrm{B}^{\hat b}]\left(\frac{\hat a}{\hat a+\hat b}(\pi_{\mathrm{A}}(a-1,b)-\pi_{\mathrm{A}}(a,b)) + \frac{\hat b}{\hat a+\hat b}(\pi_{\mathrm{B}}(a,b-1)-\pi_{\mathrm{B}}(a,b))\right)\Bigg]\\
&\quad + (1-p)\Bigg[\sum_{\substack{(\hat a,\hat b)\in\mathbb{N}^2\\ \hat a+\hat b\le K}} \eta_{\mathrm{A}}(\hat a,\hat b)\Bigg(\sum_{\substack{0\le\alpha\le\hat a\\ 1\le\beta\le\hat b\\ \dots}} (1+\delta_{\hat a,a-\beta}\delta_{\hat b,b+\beta})[\mathrm{A}^{\hat a-\alpha}\mathrm{B}^{\hat b-\beta}(\mathrm{A}^{\alpha}\mathrm{B}^{\beta})\mathrm{A}^{a-\alpha-\beta}\mathrm{B}^{b}] - \sum_{\substack{0\le\alpha\le\hat a\\ 1\le\beta\le\hat b\\ \dots}} (1+\delta_{\hat a,a}\delta_{\hat b,b})[\mathrm{A}^{\hat a-\alpha}\mathrm{B}^{\hat b-\beta}(\mathrm{A}^{\alpha}\mathrm{B}^{\beta})\mathrm{A}^{a-\alpha}\mathrm{B}^{b-\beta}]\Bigg)\\
&\quad + \sum_{\substack{(\hat a,\hat b)\in\mathbb{N}^2\\ \hat a+\hat b\le K}} \eta_{\mathrm{B}}(\hat a,\hat b)\Bigg(\sum_{\substack{1\le\alpha\le\hat a\\ 0\le\beta\le\hat b\\ \dots}} (1+\delta_{\hat a,a+\alpha}\delta_{\hat b,b-\alpha})[\mathrm{A}^{\hat a-\alpha}\mathrm{B}^{\hat b-\beta}(\mathrm{A}^{\alpha}\mathrm{B}^{\beta})\mathrm{A}^{a}\mathrm{B}^{b-\alpha-\beta}] - \sum_{\substack{1\le\alpha\le\hat a\\ 0\le\beta\le\hat b\\ \dots}} (1+\delta_{m,a}\delta_{\hat b,b})[\mathrm{A}^{\hat a-\alpha}\mathrm{B}^{\hat b-\beta}(\mathrm{A}^{\alpha}\mathrm{B}^{\beta})\mathrm{A}^{a-\alpha}\mathrm{B}^{b-\beta}]\Bigg)\\
&\quad + \sum_{1\le\alpha\le a-1} \eta_{\mathrm{A}}(a-\alpha,\alpha)[\mathrm{A}^{a-\alpha}\mathrm{B}^{\alpha}]\,\delta_{b,0} + \sum_{1\le\beta\le b-1} \eta_{\mathrm{B}}(\beta,b-\beta)[\mathrm{A}^{\beta}\mathrm{B}^{b-\beta}]\,\delta_{a,0} - [\mathrm{A}^{a}\mathrm{B}^{b}]\,\mathbb{1}_{\mathbb{N}^2\cap\bar B^1_K(0)}(a,b)\Bigg] \qquad \text{for } a,b\in\mathbb{N},\ a+b\le K.
\end{aligned}
\right.
$$

HYPERGRAPH ADAPTIVE VOTER MODEL: MEAN-FIELD MAGNETISATION

$$\dot{\mu} = \frac{2(1-p)}{N} \sum_{\substack{(\hat{a},\hat{b})\in\mathbb{N}^2 \\ \hat{a}+\hat{b}\leq K}} [\mathrm{A}^{\hat{a}}\mathrm{B}^{\hat{b}}](\hat{a}\,\eta_\mathrm{B}(\hat{a},\hat{b}) - \hat{b}\,\eta_\mathrm{A}(\hat{a},\hat{b}))$$

Proportional voting

Since $\eta_\mathrm{A}(\hat{a},\hat{b}) = \frac{\hat{a}}{\hat{a}+\hat{b}}$ and $\eta_\mathrm{B}(\hat{a},\hat{b}) = \frac{\hat{b}}{\hat{a}+\hat{b}}$,

$$\dot{\mu} = 0.$$

In equilibrium, there may be only **local consensus** or **no consensus** at all.

Majority voting

Since $\eta_\mathrm{A}(a,b) = \Theta(a-b)$ and $\eta_\mathrm{B}(a,b) = \Theta(b-a)$,

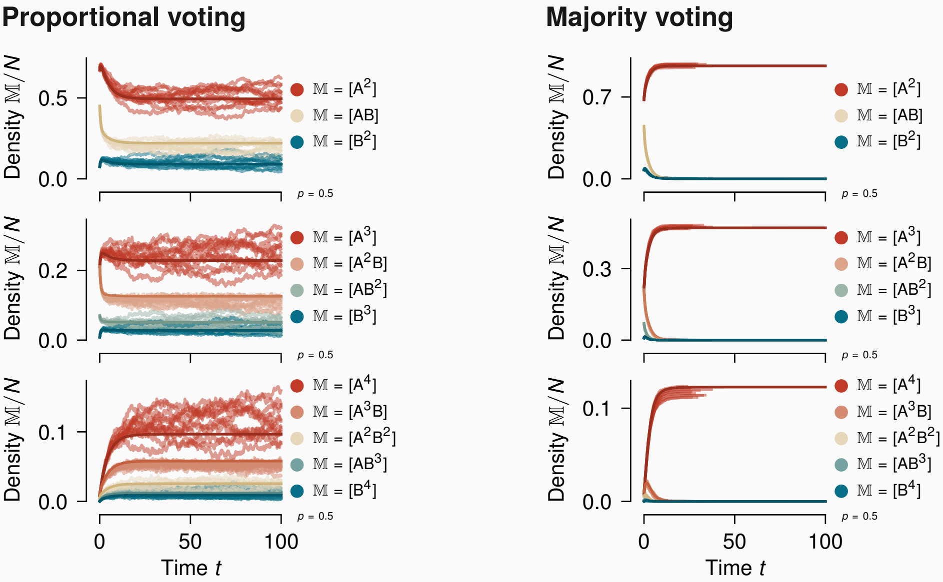
$$\dot{\mu} = \frac{2(1-p)}{N} \sum_{\substack{\sigma\in\mathbb{N}^2, |\sigma|\leq K \\ \sigma_1 > \sigma_2}} \sigma_2 \left([\mathrm{A}^{\sigma_2}\mathrm{B}^{\sigma_1}] - [\mathrm{A}^{\sigma_1}\mathrm{B}^{\sigma_2}]\right).$$

If $\mu \overset{(>)}{<} 0$, $[\mathrm{A}^{\sigma_2}\mathrm{B}^{\sigma_1}] - [\mathrm{A}^{\sigma_1}\mathrm{B}^{\sigma_2}] \overset{(>)}{<} 0$ and hence

$$\dot{\mu} \overset{(>)}{<} 0.$$

In equilibrium, there is always **global consensus**.

HYPERGRAPH ADAPTIVE VOTER MODEL: MEAN-FIELD TRAJECTORIES



CONCLUSIONS

→ We have argued that it may be crucial to take **group dynamics** into account when studying opinion formation.

→ We have shown how to define a **proper generalisation** of the classical adaptive voter model and presented a mean-field description.

→ We have discussed how different group dynamics may affect **consensus formation**.

A. Golovin, J. Mölter, and C. Kuehn (2024). "Polyadic Opinion Formation: The Adaptive Voter Model on a Hypergraph". **Ann. Phys. (Berlin) 536**:2300342. DOI: 10.1002/andp.202300342. arXiv: 2308.03640 [physics.soc-ph]

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Thank you!

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